VELOCITY PROFILES AND UNSTEADY PIPE FRICTION IN TRANSIENT FLOW

By Bruno Brunone,[1] Bryan W. Karney,[2] Member, ASCE, Michele Mecarelli,[3] and Marco Ferrante[4]

ABSTRACT: Transient conditions in closed conduits have traditionally been modeled as 1D flows with the implicit assumption that velocity profile and friction losses can be accurately predicted using equivalent 1D velocities. Although more complex fluid models have been suggested, there has been little direct experimental basis for selecting one model over another. This paper briefly reviews the significance of the 1D assumption and the historical approaches proposed for improving the numerical modeling of transient events. To address the critical need for better data, an experimental apparatus is described, and preliminary measurements of velocity profiles during two transient events caused by valve operation are presented. The velocity profiles recorded during these transient events clearly show regions of flow recirculation, flow reversal, and an increased intensity of fluid turbulence. The experimental pressures are compared to a water hammer model using a conventional quasi-steady representation of head loss and one with an improved unsteady loss model, with the unsteady model demonstrating a superior ability to track the decay in pressure peak after the first cycle. However, a number of details of the experimental pressure response are still not accurately reproduced by the unsteady friction model.

INTRODUCTION

For many years, the preoccupation of transient modelers has been to predict the peak pressures associated with a few ''worst-case'' loadings. That is, various events such as valve closures and pump failures were investigated to determine which actions resulted in the largest positive pressures and the lowest negative pressures. Such an approach was justified by the purpose of the analysis, which was to determine if the pipe system was sufficiently strong and/or the surge suppression equipment was adequately sized. Even if the transient model was conservative in the sense of overestimating the severity of these transient pressures, modelers tended to feel justifiably satisfied with their efforts, as an overestimation resulted most commonly in a safer system design.

Several factors have recently challenged this complacency. First, conditions in a complex pipe network often involve many actions, and the nature of the response depends more directly on how the pressure peak decays from one cycle to the next, not simply on the peak pressures for the first cycle. Second, there have been exciting developments in the area of inverse transient analysis in which the system's transient response is used to calibrate a numerical model of the pipe properties. These inverse methods appear to promise a simple and convenient method for determining the friction factors (as well as other parameters such as demands, pipe wave speeds, and device characteristics) for the system as a whole (Nash and Karney 1999). However, inverse transient analysis requires accurate measurements and an accurate numerical model that mimics the key physical processes in the actual network. If the performance of the numerical model is not sufficiently accurate after the first cycle of response, including the rate of decay of the transient pressure response to a new steady state, much of the potential of the inverse method may go unrealized.

Although the previous two motivations are reason enough for concern about the velocity profile and the role of unsteady friction, there is a third, and perhaps more subtle, reason. This final motivation relates to the important role of the velocity profile (and, thus, of shear stress) in water quality modeling. This connection arises from the microstructure of the turbulent flow, which results in efficient flow mixing, particularly in regions of high shear stress. The mixing process is perhaps most obvious in accelerated rates of energy loss, but the same processes that efficiently exchange momentum also generate intense velocity gradients that effectively transport material between the bulk flow and the wall. In particular, the similarity argument for turbulent exchange [e.g., Tritton (1988)] would suggest that the transfer resistance to exchange between the bulk fluid and the wall will decrease dramatically during periods where the transient shear stress is abnormally high. However, the high shear stress values that characterize the transient flow also can cause biofilm sloughing and efficient resuspension of particulates. Thus, it should come as no surprise that water quality problems have often been observed following transient events (Karney and Brunone 1999). Moreover, many current water quality models, which are typically both hydraulically steady and 1D in structure, may be missing an important transport mechanism.

The current paper briefly reviews the various approaches that have been developed for characterizing unsteady friction factors. However, a critical lack in this area has not been the creativity of modelers to propose numerical approaches, but rather the general lack of experimental data to form the basis of model selection. As a preliminary contribution for addressing this need, a laboratory setup in Italy is presented here along with a few representative experimental results (Brunone et al. 1999). The experimental results have qualitative and quantitative significance. Qualitatively, they provide a memorable impression of the nature of the velocity profile changes during an unsteady fluid flow, confirming and extending our general understanding of fluid exchange during a transient event. Quantitatively, the results confirm the need for more accurate modeling of head losses within a transient fluid flow and show what improvement is possible using a slightly more complex friction loss model. The purpose of this paper is to review and extend our understanding of fluid structure and to provide a published database that can be used to compare models of fluid structure in a transient flow.

[1]Assoc. Prof., Dept. of Civ. and Envir. Engrg., Univ. of Perugia, Perugia, Italy. E-mail: brunone@unipg.it

[2]Prof., Dept. of Civ. Engrg., Univ. of Toronto, Toronto, ON, Canada M5S 1A4; corresponding author. E-mail: karney@civ.utoronto.ca

[3]Res. Assoc., Dept. of Civ. and Envir. Engrg., Univ. of Perugia, Perugia, Italy.

[4]Asst. Prof., Dept. of Civ. and Envir. Engrg., Univ. of Perugia, Perugia, Italy.

Note. Discussion open until January 1, 2001. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on March 10, 2000. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 126, No. 4, July/August, 2000. ©ASCE, ISSN 0733-9496/00/0004-0236–0244/$8.00 + $.50 per page. Paper No. 21562.

HISTORICAL APPROACHES TO UNSTEADY FRICTION

A fundamental assumption almost universally made by modelers of transient phenomena—and indeed by almost all commercial steady pipe flow models—is that the fluid flow in a pipe is essentially 1D. That is, it is assumed that the primary characteristics of flow under both steady and unsteady conditions can be represented by the mean discharge and by variations of the mean discharge both in time and along the centerline of the pipe. Head losses due to friction, momentum changes, energy relations, and compressibility effects are thereby conventionally assumed to be directly related to the mean velocity.

Although this direct approach is strongly recommended by the simplicity of the resulting numerical models, it is important to remember that more complex models are possible and, indeed, have been attempted, particularly for transient flows. For example, Dailey et al. (1956), and later Safwat and Polder (1973), developed unsteady loss models by adding correction terms that were proportional to fluid acceleration. For laminar pipe flow, more comprehensive solutions were possible, such as Zeilke's convolution integral of temporal acceleration (Zeilke 1968). In essence, Zeilke found that weighting functions act on the velocity profile, giving the flow a ''memory'' of the historical values of the fluid acceleration. Trikha (1975) simplified these functions using exponential relations, thus improving the computational efficiency of Zeilke's model. In a somewhat related approach, Wood and Funk (1970), and later Funk and Wood (1974), looked at the impact of transient and frequency-dependent effects on system response by explicitly considering both boundary-layer effects and the recent history of energy dissipation.

More recently, Vardy et al. (1993) extended Zeilke's model to moderately turbulent, unsteady flow and then later (Vardy and Brown 1995, 1996) to turbulent flow at high Reynolds numbers. The approach was to split the flow into an outer viscosity-dominated region and an inner turbulent region characterized by uniform velocity. Other researchers have proposed different models. Suo and Wylie (1989) proposed a frequency-dependent friction factor model for application with the impedance method. Jelev (1989) argued that it was reasonable to assume that energy dissipation is proportional to internal forces in the liquid and at the pipe wall, but with a quarter-period phase shift. Motivated by experimental results, Brunone and Greco (1990) and Brunone et al. (1991a,b) assumed unsteady shear was proportional to the mean local and convective accelerations of the fluid.

Other modelers have attempted to directly resolve the fluid structure and changes in the velocity profile. For example, Vardy and Hwang (1991) derived a quasi-2D model by splitting the flow into interacting concentric annuli. Silva-Araya and Chaudhry (1997) numerically evaluated instantaneous velocity profiles to determine the ratio of the unsteady to quasi-steady energy dissipation that was then used to increase the magnitude of the steady-state friction term.

Most experimental evidence for deciding which of these models has the appropriate balance between simplicity and accuracy has been anecdotal at best. In fact, many models have been proposed or defended on the basis of their reasonableness alone, and others by merely looking at the rates of decay of a pressure signal. What is needed is a more comprehensive set of measurements of pressure and velocity to begin to sort out the true nature of the evolution of the fluid structure under transient conditions. The first step in this direction is discussed in the next section.

EXPERIMENTAL PROCEDURE

The experimental setup for measuring the velocity profile and pressure decay in a pipe system is briefly described in this section. In general terms, the apparatus is simple, consisting of a length of pipe connected to a control valve that is used to rapidly adjust the flow and a set of sensitive instruments that are used to record the pipe's response to transient events. The most unusual aspect of the apparatus is the velocity probe that is used to measure essentially instantaneous velocity values across the width of the pipe.

More specifically, the apparatus comprises a 352-m length of polyethylene pipe with an internal diameter of 93.8 mm and wall thickness of 8.1 mm. To obtain a substantial length within the confined lab, the pipe is arranged concentrically with bends having a minimum radius equal to 1.5 m, and, except for the last short part, it is almost horizontal (Fig. 1). The large radius of curvature was selected to reduce secondary currents in the pipe system. An air vessel is used as a supply reservoir because it keeps upstream pressures essentially constant. The prescribed pressure is also maintained by varying the speed of two submerged pumps drawing water from the recycling reservoir.

A ball valve at the far end of the pipeline is used to create controlled opening and closing actions by adjusting the discharge into a free surface tank. Pressure and velocity profiles are measured at the downstream end section (measurement section #1) and a section at a distance of about 24 m from the air vessel (measurement section #2). Pressure transducers are of the strain-gauge type, with a recording range of 0 to 100 m of water, an accuracy of 0.5% of the full-range scale, and a response time of 50 ms. The pressure datum is taken for convenience as the floor of the laboratory.

A DOP1000 ultrasonic velocimeter samples information about the local characteristics of the velocity field during transient conditions; its working principle, as sketched in Fig. 2, is based on the Doppler effect, detecting and processing the echoes of ultrasonic pulses reflected by particles in the flowing liquid (*DOP1000* 1997; Lemmin and Rolland 1997). In summary, a short train of sinusoidal waves with frequency f_e is emitted from the transducer and then repeated at a lower frequency. Between these two pulses, the same transducer receives the waves reflected back by particles moving with the fluid (the so-called monostatic configuration). The measurement of the time of travel to a certain depth or distance across the pipe gives the position of the scattering volume. By measuring the Doppler frequency shift f_d at different times, it is possible to obtain an almost instantaneous velocity profile using the relation $v = cf_d/2f$, where v is the velocity component in the direction of the ultrasonic beam, and c is the speed of sound in the liquid.

The resolution of this approach to distinguish signals and velocity components is defined by the size and shape of the sampling volume, and it is typically a few cubic millimeters. The minimum value of the acquisition time is about 3 μs. To improve the quality of the measured velocity profiles, the standard layout with the probe placed externally (with acoustic continuity maintained with a gel) was replaced by the arrangement shown in Fig. 2. In the modified arrangement, the probe is positioned to be in direct contact with the flowing fluid; however, because care is taken to avoid disturbances in the flow, the adopted technique is essentially nonintrusive.

EXPERIMENTAL RESULTS

Various experiments representing different initial values of flow, different value actions (e.g., opening versus closing), and different closure times have been conducted with the apparatus described in the previous section. For reasons of space and time, not all of these results can be discussed here. Instead, two representative experiments are discussed in greater detail to illustrate the significance and role of unsteady friction and the velocity profile on transient events.

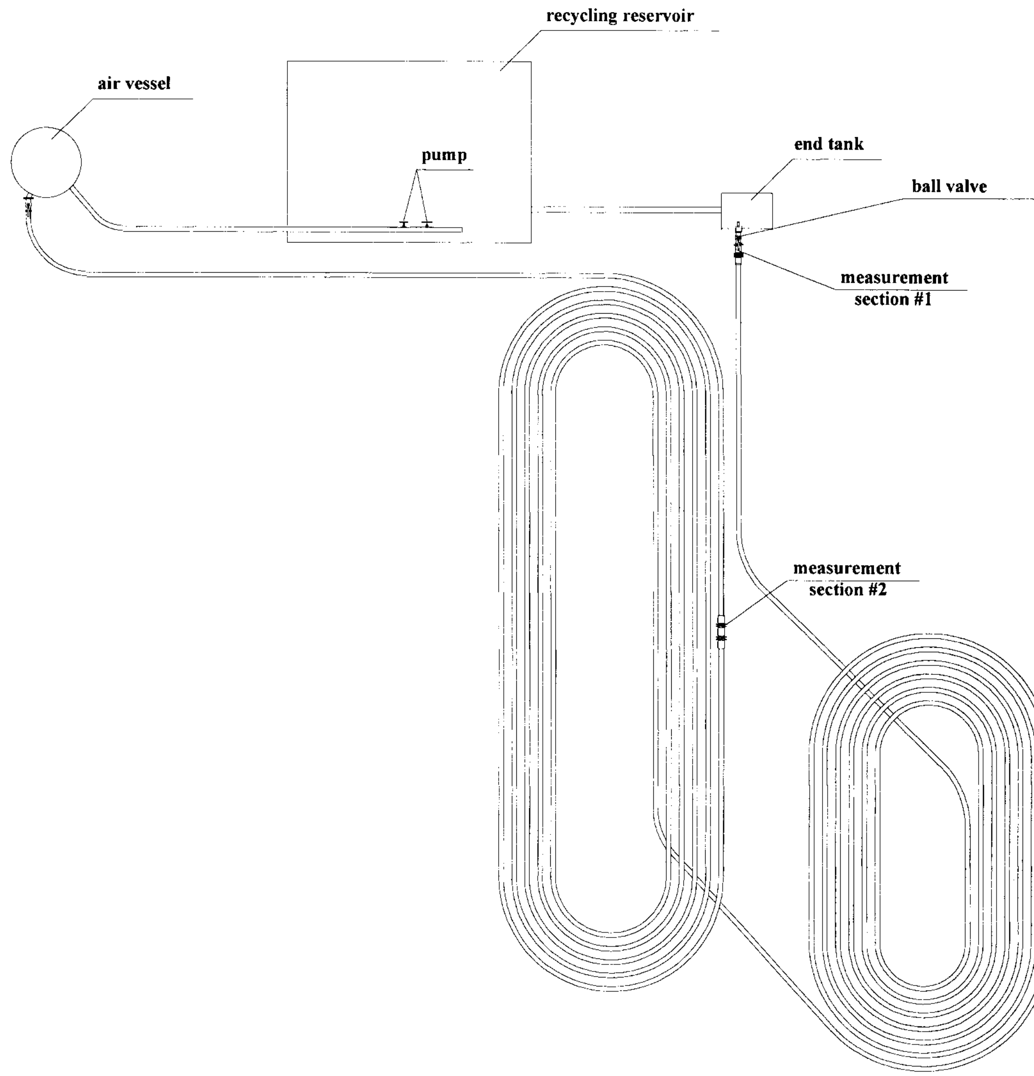

FIG. 1. Experimental Setup (Plan View)

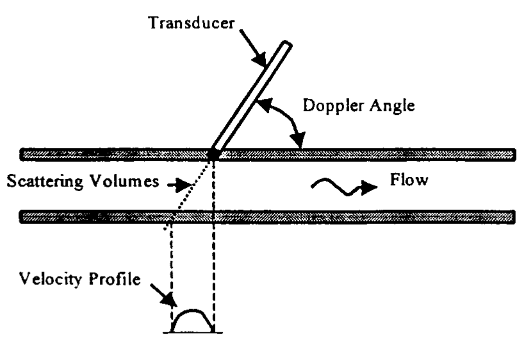

FIG. 2. Working Principle of Ultrasonic Velocimeter [Modified from *DOP1000 User's Manual* (*DOP1000* 1997)]

Test Cases

Fig. 3 gives the traditional response of a pressure transducer, adjusted to provide the piezometric head at the valve end and, in the lower portions of the plots, eight corresponding velocity profiles during the transient event. The initiation time in this and subsequent plots is arbitrary, indicating only the passage of time from the start of data acquisition. The points labeled with small numbers on the time history in the upper plot label the instants corresponding to the displayed velocity profiles.

The transient event associated with Fig. 3 has the following characteristics: the initial discharge in the line is 4.93 L/s, and the valve is operated from a fully open to a fully closed position in 0.06 s. The time history of the pressure and the velocities are measured at the downstream end of the system, just upstream from the valve.

The second test is summarized in the extended plot of Fig. 4. This second transient event has the following characteristics: the initial discharge in the line is slightly larger at 6.43 L/s, and the valve is operated from fully open to closed in a longer duration of 0.315 s. Both the time history of the pressure and the velocities are measured at 328 m upstream from the control valve.

The two different cases were chosen to provide some variation in initial Reynolds number as well as a marked difference in the duration of the deceleration phase. The different points on the pipe system, owing to the varying impact of wave reflections form the ends of the pipe system, also provide a changing perspective on the transient event. Clearly, test 2 with the longer closure time and the measurements at the midpoint (Fig. 4) would be expected to have a smoother rise to the maximum pressure followed by a quicker return to near steady pressure values owing to the more rapid pressure wave reflection propagating back from the air vessel.

General Characteristics of Response

Relative to conventional modeling, one of the first observations that can be made from the plots in Figs. 3 and 4 is the rapid decay and smoothing of pressure peaks after the end of a complete valve closure; quite smooth curves are obtained

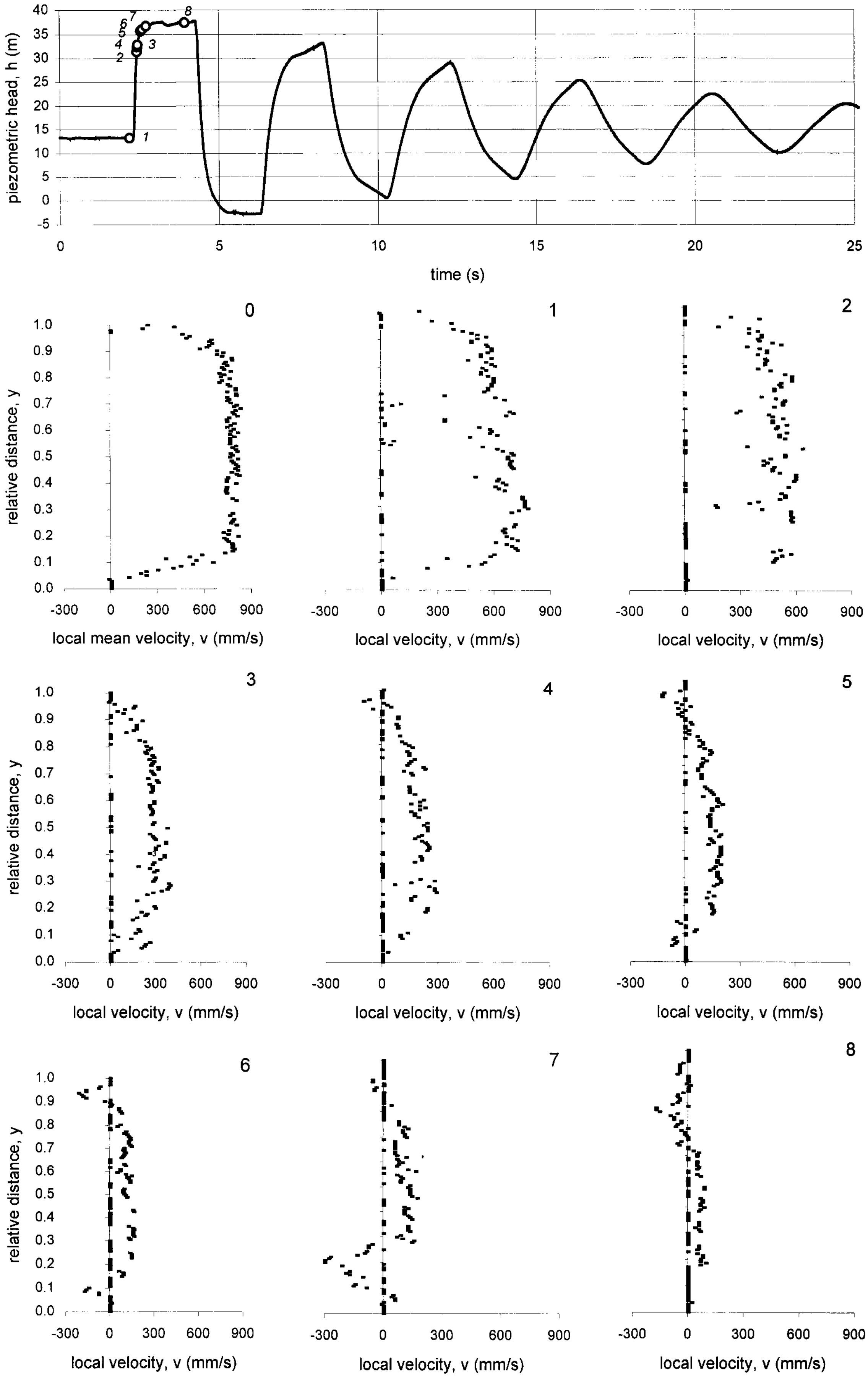

FIG. 3. Test 1 Experimental Results Showing Pressure Time History and Associated Velocity Profiles

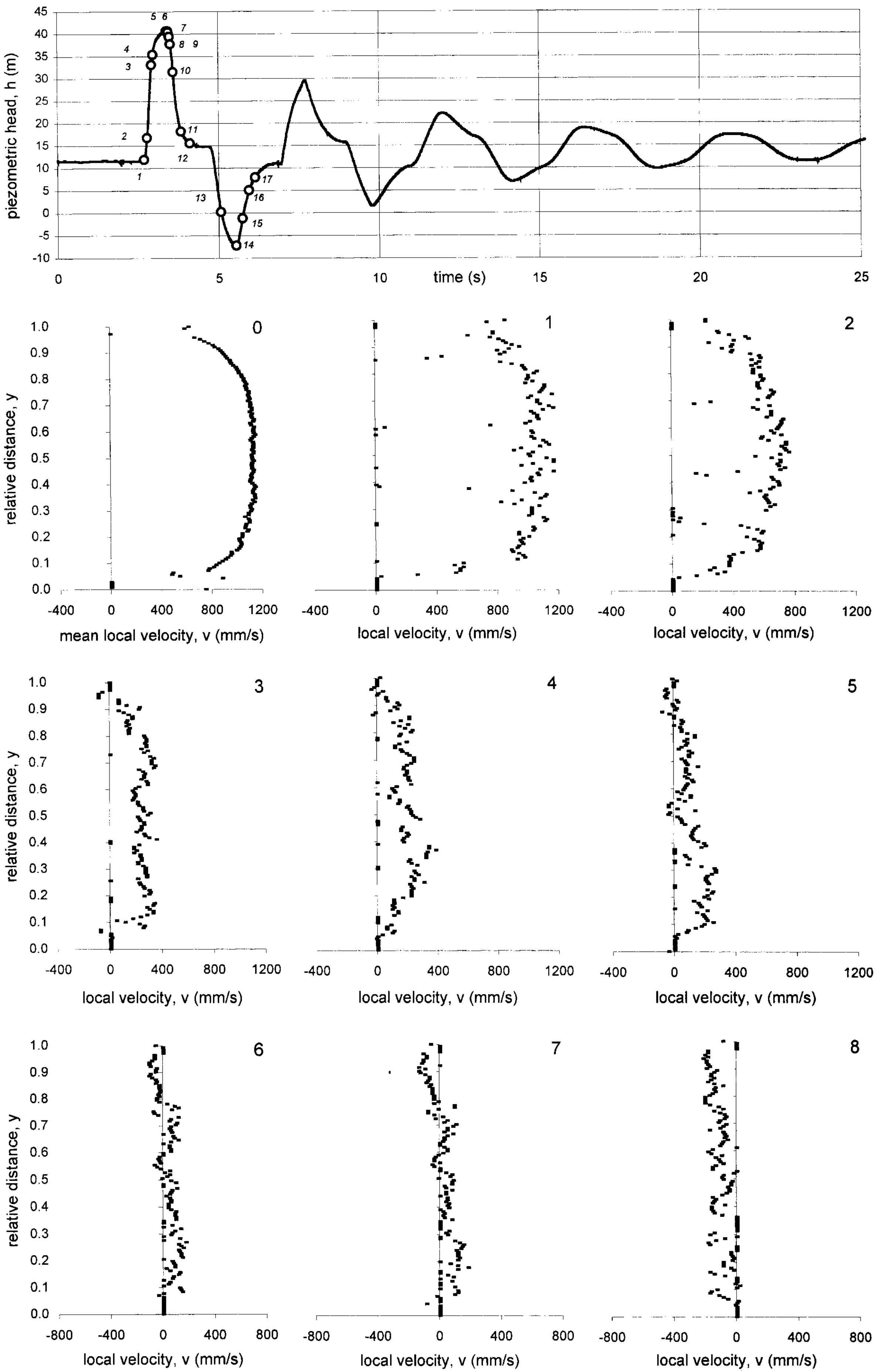

FIG. 4. Test 2 Experimental Results Showing Pressure Time History and Associated Velocity Profiles

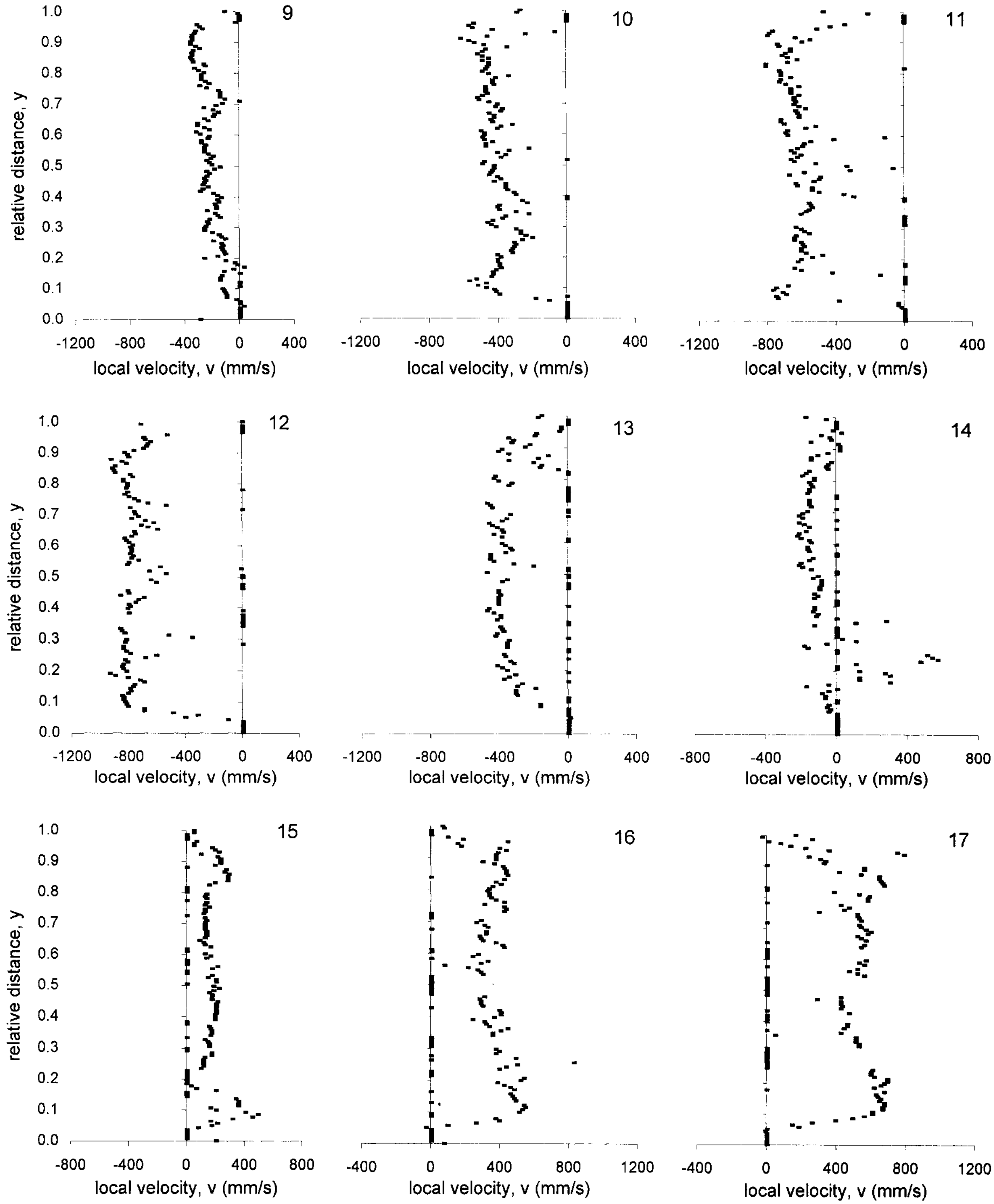

FIG. 4. (*Continued*)

after only three or full wave cycles. More profoundly, however, the plots clearly illustrate the connections between the velocity and pressure waves traveling in the pipe system. For example, Fig. 3 clearly shows the deceleration of the velocity as the high pressure wave is established at the closing valve and how the action of this pressure wave progressively brings the entire velocity profile to rest. However, the process of deceleration is clearly shown to be anything but uniform, with subplot 7 in Fig. 3 showing a dramatic example of a near zero mean flow having a highly complex internal structure. Plots like this graphically demonstrate how it is possible to obtain such high rates of energy dissipation and shear stress even in a flow having little net discharge.

Although two test results are obviously too small a sample to be definitive, the following additional comments allow a more complete interpretation of the displayed velocity profiles:

- The individual dots in Figs. 3 and 4 give the velocity of individual particles in the flow at the sampling locations. However, the points on the *y*-axis (nominally zero velocity) correspond to missing measurements because of the absence of a flowing particle in that point; they do not necessarily represent a true zero velocity and should be ignored in interpreting the plots.
- Turbulence in the flow is generally evident by some of the scatter about the average position of the profile. Unfortunately, measurement errors cannot easily be separated from the turbulent fluctuations owing to the difficulty of

repeating experiments in detail. Interestingly, even the supposed state profiles show considerable fluctuations as turbulent eddies, and fluctuations traverse the measurement section.

- Few measurement points are located in the zone close to the pipe wall. Adjustments in the experimental technique are being considered to reduce the impact of this problem.
- The behavior of the unsteady-state velocity profiles is quite different from the ones in steady-state conditions (e.g., compare the first profile in Figs. 3 and 4 to subsequent ones). Transient flows appear to create significant turbulence and irregularity in the flow profile. This effect appears particularly significant during deceleration phases.
- Clearly, there are important differences in velocity gradients at the wall between steady and unsteady conditions. This observation generally supports the notion that unsteady friction losses may differ significantly from those arising from steady flows and qualitatively confirms the work of others who have inferred such gradients.
- As has been mentioned, some phases of the flow show reversals of velocity within a single profile, with the near-wall flow moving in the opposite direction to the center of the flow. However it is just as obvious that the velocity profile can change its properties and character quickly over very short durations of time.
- Some strong asymmetries sometimes exist in the flow. For example, profile 14 of Fig. 4 indicates that forward flow is taking place only in the lower part of the pipe and a small backward flow in much of the upper portion.

The general nature of the response of the velocity profile to the transient pressure wave has a natural interpretation. The higher pressure caused by the closed valve creates an adverse pressure gradient that acts more or less uniformly over the profile. This pressure difference decelerates the flow but will tend to reverse the flow first along the walls of the pipe, owing principally to the smaller initial velocity at these locations. This behavior was anticipated by a number of previous researchers. The measured profiles show a strong resemblance to the theoretical ones of others [e.g., see the plots in Vardy and Hwang (1991) or in Silva-Araya and Chaudhry (1997)].

NUMERICAL RESULTS

Although the qualitative observations that have been made thus far are of some general interest, the question that an engineer or a modeler asks is more direct: can the basic elements of the flow be reproduced numerically? Interestingly, a relatively small amount of work is required to provide some quantitative insight into the importance of the structure of a fluid flow under unsteady conditions. A thorough review, theoretical development, discussion, and extension of these unsteady friction models such as the one discussed here can be found in a current paper by Axworthy et al. (2000).

More precisely, the conventional method of characteristics in a pipe system can be used to represent the primary elements of fluid pressure, conduit properties, and fluid inertia (Chaudhry 1987; Wylie and Streeter 1993). However, an unsteady model needs to be used for evaluation of the unsteady friction factor *f*. In this work, the following expression for *f* is used (Brunone et al. 1995):

$$f = f_s + \frac{kD}{V^2}\left(\frac{\partial V}{\partial t} - a\,\frac{\partial V}{\partial x}\right) \tag{1}$$

where f_s = steady-state or Darcy-Weisbach friction factor (e.g., from the Moody diagram); V = mean flow velocity; a = pressure wave speed; D = internal diameter; x = axial coordinate; t = time, while the ''decay'' coefficient k is given by

$$\frac{y_n}{y_{n-1}} = \left(\frac{1}{1+k}\right)^2 \tag{2}$$

in which y_n and y_{n-1} = maximum piezometric heads, with respect to the steady-state value, in any two consecutive periods taken sometime after the end of the closing (Carravetta et al. 1992). In general, for a particular numerical test, k is treated as a constant allowing the numerical solution to be direct and explict. As a consequence, three parameters had to be estimated: the characteristic roughness size ε (Colebrook-White equation), a, and k on the basis of pressure and discharge data in steady-state conditions, the periodicity of pressure waves, and the damping of pressure peaks after the completion of the closing, respectively (Brunone and Morelli 1999).

The results of this modeling exercise are summarized in Fig.

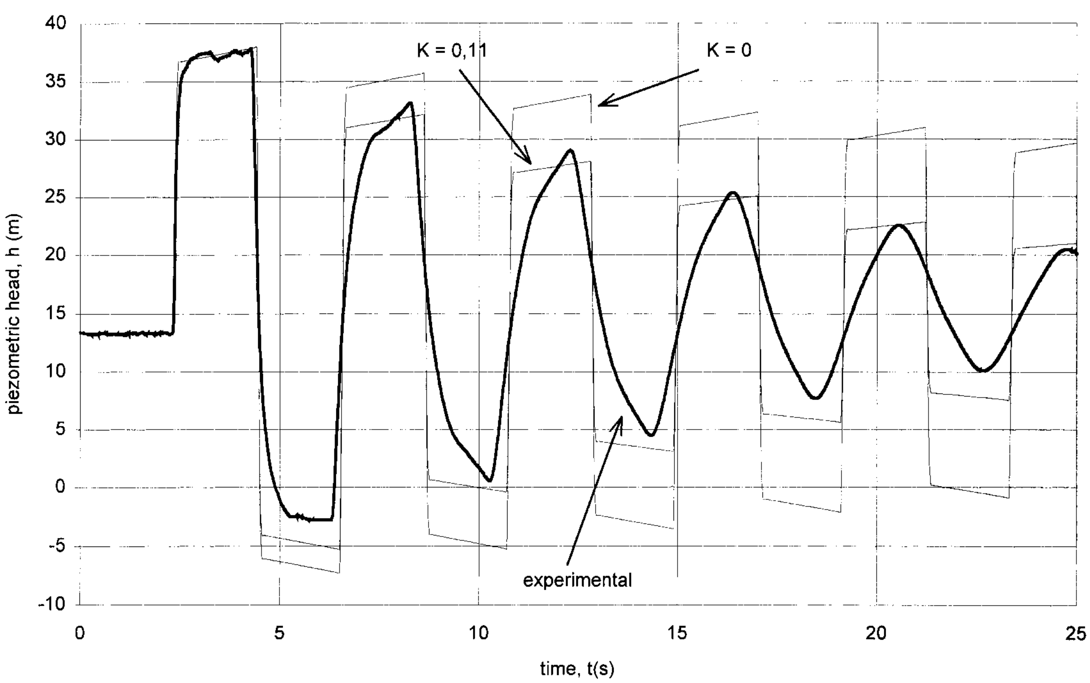

FIG. 5. Test 1 Experimental and Numerical Pressure Time Histories

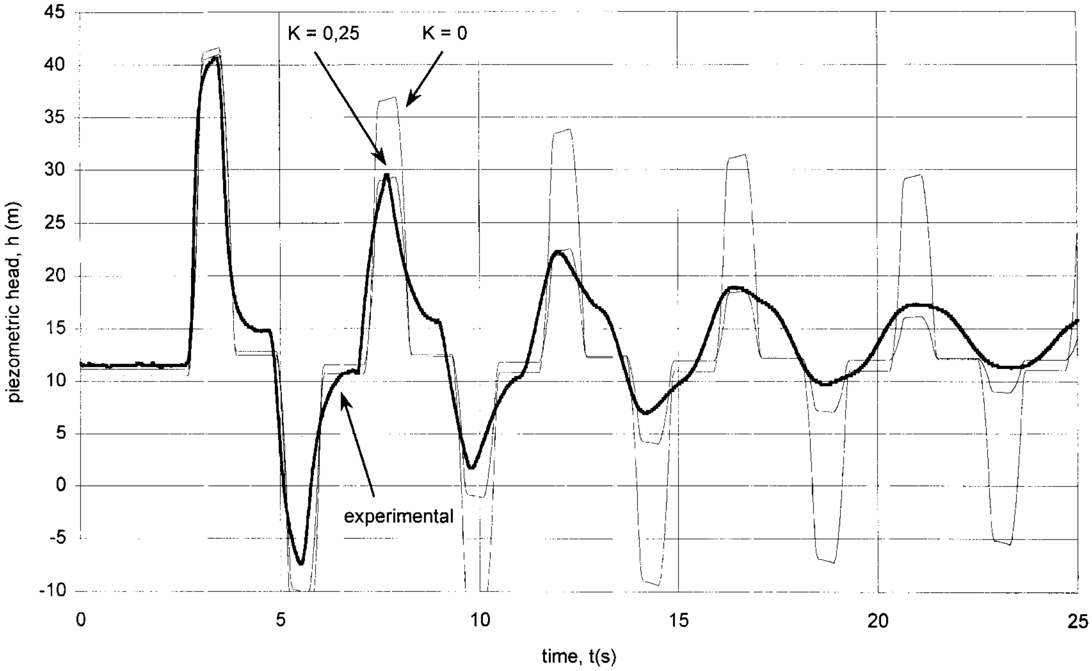

FIG. 6. Test 2 Experimental and Numerical Pressure Time Histories

5 for the first test and in Fig. 6 for the second. These figures show three curves: (1) the heavy line shows the experimental results as recorded; (2) the thin lines labeled $k = 0$ show the results that would be obtained from a conventional water hammer modeling used with steady-state frictional losses; and (3) the results of the numerical model with the value of the unsteady loss factor k indicated on the plots. A comparison of these plots shows clearly that the unsteady friction model fits the experimental results much better than a model using only steady-state losses. However, it is also fair to say that the full complexity of the experimental test is not completely reflected by even the extended loss model.

CONCLUSIONS

Unsteady flow results in many interesting and important phenomena in a pipeline system. The traditional preoccupation with maximum and minimum pressures is still relevant, but so are evolving concerns about water quality and fluid structure changes during transient events. As always, the best models arise from an optimum resolution of the tension between the simplicity of the numerical approach and the accuracy of the resulting predictions of flow behavior. With improvements in computational power, it is now possible to account numerically for a more complete set of physical terms and interactions; yet, if the resulting models are to be meaningful, they must be based on measurements and experimental data. To this end, a set of experiments relating the evolving fluid structure to a transient event are considered in this paper, with the goal eventually allowing a more adequate evaluation of the role of unsteady friction losses in pipeline systems. The results clearly demonstrate the complex nature of the transitions between flow states and the range of fluid behavior that will ultimately need to be taken into account in comprehensive numerical models.

ACKNOWLEDGMENT

This research has been partially funded by the Italian Ministry of University and Scientific Research under the project ''Whirling, turbulent and chaotic processes. Waterworks and environmental applications.''

APPENDIX I. REFERENCES

Axworthy, D. A., Ghidaoui, M., and McInnis, D. M. (2000). ''An extended thermodynamics derivation of energy dissipation in unsteady pipe flow.'' *J. Hydr. Engrg.*, ASCE, 126(4), 276–287.

Brunone, B., Golia, U. M., and Greco, M. (1991a). ''Modelling of fast transients by numerical methods.'' *Proc., Int. Conf. on Hydr. Transients with Water Column Separation*, IAHR, Delft, The Netherlands, 273–280.

Brunone, B., Golia, U. M., and Greco, M. (1991b). ''Some remarks on the momentum equation for fast transients.'' *Proc., Int. Conf. on Hydr. Transients with Water Column Separation*, IAHR, Delft, The Netherlands, 201–209.

Brunone, B., Golia, U. M., and Greco, M. (1995). ''Effects of two-dimensionality on pipe transients modeling.'' *J. Hydr. Engrg.*, ASCE, 121(12), 906–912.

Brunone, B., and Greco, M. (1990). ''Improvements in modeling of water hammer and cavitating flow in pipes. Experimental verification.'' *Proc., 22nd Convegno Nazionale di Idraulica e Costuzioni Idrauliche*, Vol. 4, 147–160 (in Italian).

Brunone, B., Karney, B. W., and Ferrante, M. (1999). ''Velocity profiles, unsteady friction losses and transient modelling.'' *Proc., 26th Annu. Water Resour. Plng. and Mgmt. Conf.*, ASCE, Reston, Va. (on CD-ROM).

Brunone, B., and Morelli, L. (1999). ''Automatic control valve-induced transients in operative pipe system.'' *J. Hydr. Engrg.*, ASCE, 125(5), 534–542.

Carravetta, A., Golia, U. M., and Greco, M. (1992). ''On the spontaneous damping of pressure oscillations in water-hammer transients.'' *Proc., 23rd Congr. of Hydr. and Hydr. Works*, Vol. 4E, 67–79 (in Italian).

Chaudhry, M. H. (1987). *Applied hydraulic transients*, Van Nostrand Reinhold, N.Y.

Dailey, J. W., Hankey, W. L., Olive, R. W., and Jordaan, J. M. (1956). ''Resistance coefficients for accelerated and decelerated flows through smooth tubes and orifices.'' *J. Basic Engrg.*, Ser. D, 78, 1071–1077.

DOP1000 user's manual. (1997). Signal Processing s.a., Zurich, Switzerland.

Funk, J. E., and Wood, D. J. (1974). ''Frequency response of fluid lines with turbulent flow.'' *J. Fluids Engrg.*, 96, 365–369.

Jelev, I. (1989). ''The damping of flow and pressure oscillations in water hammer analysis.'' *J. Hydr. Res.*, Delft, The Netherlands, 27(1), 91–114.

Karney, B. W., and Brunone, B. (1999). ''Water hammer in pipe networks: Two case studies,'' *Water industry systems: Modelling and optimization applications*, D. Savic and G. Walters, eds., Vol. 1, Research Studies Press Ltd., Baldock, England, 363–376.

Lemmin, U., and Rolland, T. (1997). ''Acoustic velocity profiler for laboratory and field studies.'' *J. Hydr. Engrg.*, ASCE, 123(12), 1089–1098.

Nash, G. A., and Karney, B. W. (1999). ''Efficient inverse transient analysis in series pipe systems.'' *J. Hydr. Engrg.*, ASCE, 125(7), 761–764.

Safwat, H. H., and Polder, J. (1973). ''Friction-frequency dependence for oscillatory flows in circular pipes.'' *J. Hydr. Div.*, ASCE, 99(11), 1933–1945.

Silva-Araya, W. F., and Chaudhry, M. H. (1997). ''Computation of energy dissipation in transient flow.'' *J. Hydr. Engrg.*, ASCE, 123(2), 108–115.
Suo, L., and Wylie, E. B. (1989). ''Impulse response method for frequency-dependent pipeline transients.'' *J. Fluids Engrg.*, 111(4), 478–483.
Trikha, A. K. (1975). ''An efficient method for simulating frequency-dependent friction in transient liquid flow.'' *J. Fluids Engrg.*, 97(1), 97–105.
Tritton, D. J. (1988). *Physical fluid dynamics*, 2nd Ed., Oxford University Press, Oxford, U.K.
Vardy, A. E., and Brown, J. M. B. (1995). ''Transient, turbulent, smooth pipe friction.'' *J. Hydr. Res.*, Delft, The Netherlands, 33(4), 435–456.
Vardy, A. E., and Brown, J. M. B. (1996). ''On turbulent, unsteady, smooth-pipe friction.'' *Proc., 7th Int. Conf. on Pressure Surges and Fluid Transients in Pipelines and Open Channels*, BHR Group Ltd., Harrogate, England, 289–311.
Vardy, A. E, and Hwang, K. L. (1991). ''A characteristic model of transient friction in pipes.'' *J. Hydr. Res.*, Delft, The Netherlands, 29(5), 669–684.
Vardy, A. E., Hwang, K. L., and Brown, J. M. B. (1993). ''A weighting function model of transient turbulent pipe friction.'' *J. Hydr. Res.*, Delft, The Netherlands, 31(4), 533–548.
Wood, D. J., and Funk, J. E. (1970). ''A boundary-layer theory for transient viscous losses in turbulent flow.'' *J. Basic Engrg.*, 92, 865–873.
Wylie, E. B., and Streeter, V. L. (1993). *Fluid transients in systems*, Prentice-Hall, Englewood Cliffs, N.J.
Zeilke, W. (1968). ''Frequency-dependent friction in transient pipe flow.'' *J. Basic Engrg.*, Ser. D, 90(1), 109–115.

APPENDIX II. NOTATION

The following symbols are used in this paper:

a = pressure wave speed (m/s);
c = acoustic wave speed (m/s);
D = internal pipe diameter (m);
f = unsteady-state friction factor;
f_d = Doppler frequency shift (Hz);
f_e = transducer (emitted) frequency (Hz);
f_s = steady-state (Darcy-Weisbach) friction factor;
k = unsteady friction decay coefficient;
h = piezometric head (m);
t = time (s);
V = mean velocity (m/s);
v = local velocity (m/s);
x = axial distance (m);
y = relative distance transverse to pipe axis;
y_i = maximum piezometric head in *i*th period (m); and
ε = characteristic roughness size (m).